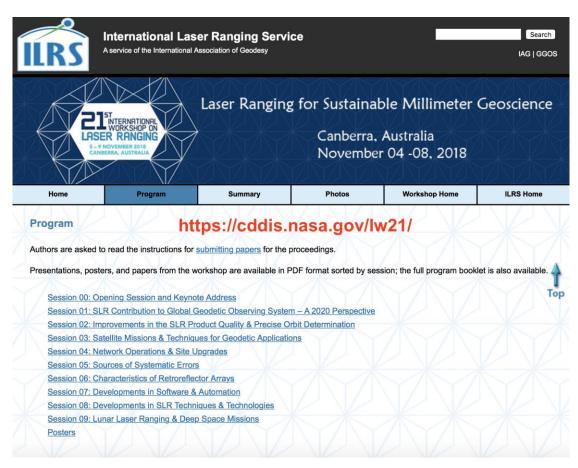
## Thermal-Optical Design of a Geodetic Satellite for One Millimeter Accuracy

by
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#### Contents of full paper:

#### Abstract

- 1. Basic Principles
- 2. Centroid vs Incidence Angle
- 3. Thermal Problems
- 4. Polarization Asymmetry
- 5. Centroid vs Velocity Aberration
- 6. Thermal Simulations
- 7. Equations of Equilibrium
- 8. Temperatures of Core and Cubes
- 9. Summary

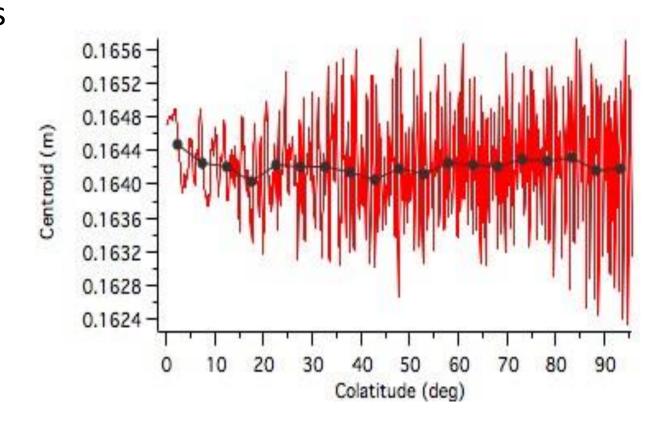


### Need For a New Design

- The design goal for LAGEOS was 5 mm
- The present design goal is 1 mm
- Problems with LAGEOS:
  - Cubes are too large for the velocity aberration
  - The use of dihedral angle offsets creates a complicated pattern
  - Thermal gradients increase sharply with CCR size
  - Thermal problems are the major uncertainty for current satellites

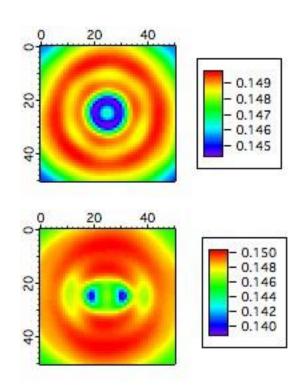
# Section 2. Centroid vs Colatitude using 1.0 inch CCRs

- The centroid is computed as the incidence angle spirals around the satellite from the North pole
- The peak to peak variation for different incidence angles (red) is 3.4 mm
- The peak to peak variation by averaging over 5° intervals in colatitude (black) is .4 mm

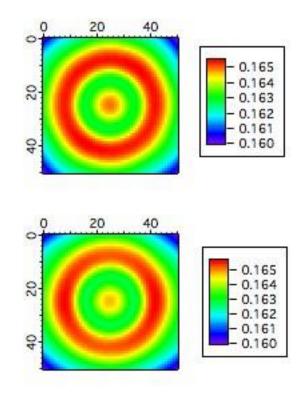


# Section 4 Asymmetry vs Size and Polarization

Circular/ 1.5 inch CCR Linear

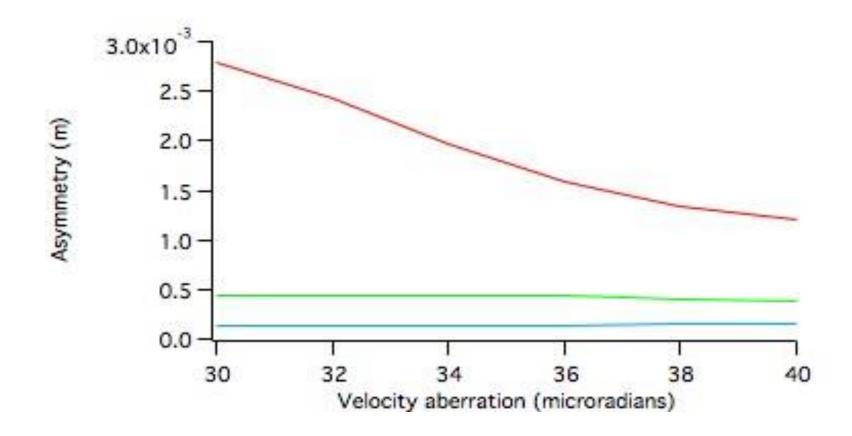


Circular/ 1.0 inch CCR Linear



## Section 4 Comparison of the Asymmetry in the Centroid

Red = 1.5 inch linear, Green = 1.0 inch linear, Blue = 1.0 inch circular

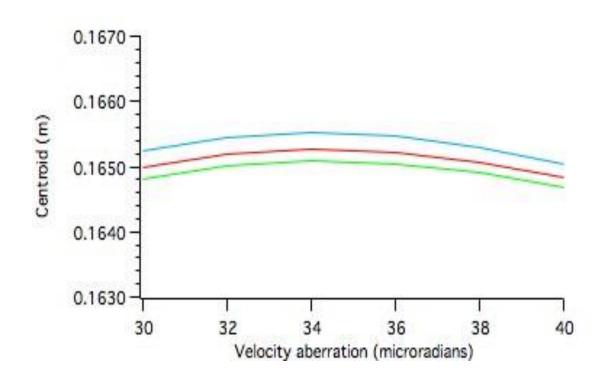


# Section 5. Centroid vs Velocity Aberration Red = average, Green = min, Blue = max

#### 1.5 inch cube

## 0.1490 0.1480 0.1470 0.1460 30 32 34 36 38 40 Velocity aberration (microradians)

#### 1.0 inch cube

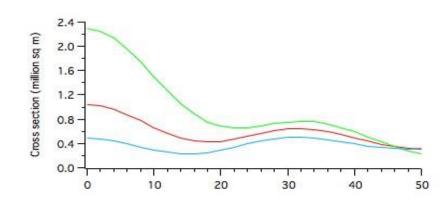


Section 6. Effect of Thermal Gradient on Cross Section for Positive (blue) and Negative (green) Dihedral Angle Offset, Red is No Thermal Gradient.

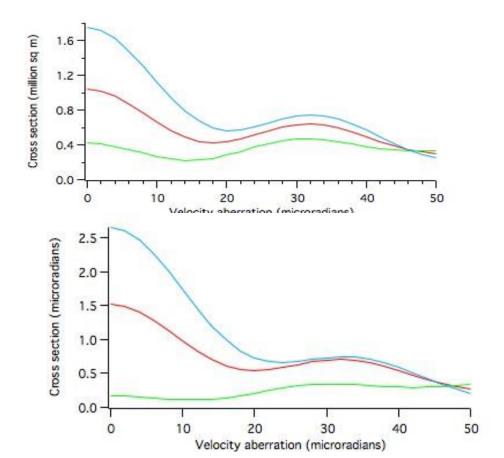
Low emissivity

Case 11 Floating mount/Case 12 with conduction

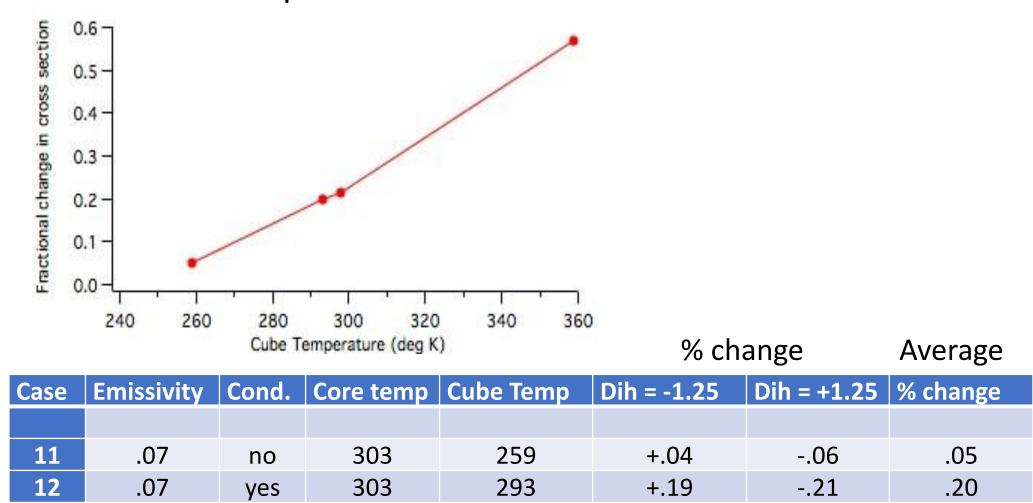
1.6 1.4 - 1.2 - 1.0 - 0.8 - 0.6 - 0.4 - 0.2 - 0.0 - 0.2 - 0.0 - 0.2 - 0.0 Velocity aberration (microradians)



High emissivity
Case 17 Temp 299 deg/Case 16 temp 360 deg



## Section 6. Fractional Change in Cross Section vs Temperature of the Cube Corner



298

359

-.27

-.51

343

413

**17** 

**16** 

.29

.29

no

no

.22

.57

+.16

+.06

Section 7. Equation for the equilibrium temperature of the core of the satellite

$$t_{core}^{4} = \frac{\frac{Ne_{eff}A_{b}H_{cube}}{e_{eff}A_{b} + e_{cube}A_{f}} + H_{core}}{S[e_{core}A_{core} + Ne_{eff}A_{b}(\frac{e_{cube}A_{f}}{e_{eff}A_{b} + e_{cube}A_{f}})]}$$

### Section 8. Calculation of the Core and Cube Temperature

Col	1	2	3	4	5	6	7	8	9	10	11
Case	⟨ Core	$\sum_{core}$	$\sum_{Cavity}$	$t_{sphere}$	$t_{core}$	$t_{cube}$	$H_{core}$	$H_{cube}$	$R_{core}$	R <sub>to Cube</sub>	$R_{cube}$
1	.62	.29	.05	338.7	327.6	209.1	75.92	.0177	66.4	.0317	.0494
2	.62	.29	.29	338.7	302.7	252.5	75.92	.0177	49.7	.0874	.1050
3	.15	.80	.05	184.3	183.3	164.6	18.37	.0176	18.4	.0013	.0190
4	.15	.80	.29	184.3	181.0	170.7	18.37	.0177	17.1	.0043	.0220

#### **Column:**

- 1. Solar absorptivity of the core
- 2. Emissivity of the core
- 3. Emissivity of the cavity
- 4. Temperature of a sphere with no cube corners
- 5. Temperature of the core

- 6. Temperature of a cube
- 7. Solar heating of the core
- 8. Solar heating of a cube corner
- 9. Thermal radiation from the core
- 10. Thermal radiation to a cube corner
- 11. Radiation from the front face of a cube corner

### Section 9. Summary

- 1. 1" CCRs provide more uniform surface coverage and less incidence angle variations.
- 2. 1.5" CCRs are too large for the velocity aberration and required dihedral angle offsets (DAO). This produces a "lumpy" diffraction pattern that causes variations in range within the far field diffraction pattern.
- 3. There is an interaction between DAO and the phase changes due to total internal reflection producing an asymmetrical diffraction pattern with linear polarization.
- 4. 1" CCRs provide the necessary beam spread to account for velocity aberration without the need for DAO. This also removes the asymmetry in the diffraction pattern with linear polarization.
- 5. The diffraction pattern without DAO is smoother than the patterns with DAO.
- 6. The diffraction pattern of an uncoated cube has a ring of spots around the central peak. The size of the cube can be chosen to put the velocity aberration on this ring of spots rather than on a slope in the diffraction pattern. This reduces the variation of the range correction with velocity aberration. This ring of spots is a very stable part of the diffraction pattern that does not change much due to various perturbations.

### Section 9. Summary (cont.)

- 7. Thermal effects increase as some power (~4th power) of the size. The reduction in size from 1.5" to 1" appears to reduce variations in the cross section by a factor of ~5-6
- 8. Eliminating the DAO allows the use of COTS (Commercial Off-The-Shelf) CCRs that are inexpensive and readily available. Testing by Ludwig Grunwaldt showed that the optical quality of these cubes is as good as custom made cubes with DAO that are expensive and time consuming to manufacture.
- 9. There are small unintentional DAO in COTS cubes that are generally less than one arsec but can be up to two arcsec. The effect of a positive (>90°) offset is in the opposite direction from the effect of a negative (<90°) offset. Since the mean offset is zero the positive offsets tend to partially cancel the effect of the negative offsets.
- 10. Thermal simulations show that the effect of thermal gradients in a 1" cube is very small with a floating mount.
- 11. A floating mount requires leaving a small gap between the ring and the cube. This could potentially result in damage to the cube due to vibrations during launch. Vibration testing with a very large gap showed no damage to the cube.
- 12. The thermal simulations show that the fractional change in cross section due to thermal gradients is nearly linear with the temperature of the cube.